REMARKS ON THE COMPRESSOR - ACCELERATOR INTERFACE

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A. Introduction

In a conventional electron linear accelerator, electrons emerge from a gun as a DC-beam to enter a bunching section which causes a substantial fraction of them to traverse the main accelerator at an asymptotic phase close to the crest of the accelerating wave. The buncher is phased from the accelerator so that the time structure of the beam is established by the accelerator itself and a high degree of uniformity can be achieved in phase and final energy of the electrons during the more or less steady state condition within a pulse, and from pulse to pulse. In the ERA, there is no steady state condition, there is no natural relation between the time sequence of events in compression, release, and timing of the accelerator and there is even uncertainty in the charge and mass of the ring. There is clearly a need for devices to correlate the systems; in order to enter an r.F. structure within, say, a 10° phase range requires a precision of 1.5×10^{-10} sec. at 200 mc and 2.5×10^{-11} sec. at 1200 mc, whereas even the jitter in timing release from the compressor is likely to be $5 - 10 \times 10^{-9}$ secs.

Two types of accelerators are being considered, an r.F. linac and an induction, or pulsed line, accelerator. In either case, the synchronization problem can be attacked by an action on the released ring or by a readjustment of the accelerator on the basis of information about the ring. The initial speed of the ring is given by the formula for the effect of adiabatic changes, expressed for this purpose in the following form:

$$\beta_R = \beta_{cp}\sqrt{\frac{B_{cp} - B_R}{B_{cp}}} \tag{1}$$

Where β_R is the longitudinal velocity after release and B_R and B_{cp} are the magnetic field strength in the accelerating column and in the compressor, respectively. Since $\beta_{cp} \sim 1$, a small release velocity would require a very small difference in fields, which might be difficult to control. It seems more reasonable to consider differences of perhaps 25%, which implies that $\beta_R \sim 0.5$. The following remarks on the four possible situations (two types of accelerator combined with two approaches for synchronization) thus are predicated on the assumption of a fairly high release velocity.

B. Pulsed Line Acceleration

If the ring sees a longitudinal electric field uniform in space and time, its momentum will increase according to:

$$\gamma\beta = (\gamma\beta)_R + \frac{QE}{M_\perp c}t \tag{2}$$

and its spatial position according to:

$$z = \frac{M_\perp c^2}{QE}\left[\gamma(t) - \gamma_R\right] = \frac{M_\perp c^2}{QE}\left[\sqrt{1 + \left((\gamma\beta)_R + \frac{QE}{M_\perp c}t\right)^2} - \sqrt{1 + (\gamma\beta)_R^2}\right] \tag{3}$$

WHERE Q and $M_\perp$ are the charge and effective mass of the ring, E is the applied field, and γ and β refer to the longitudinal motion. The variation in time of arrival at a given downstream location, Z, due to variations in $\frac{QE}{M_\perp}$, $(\gamma\beta)_R$ and starting time is obtained by differentiating eq (3):

$$\Delta t = \Delta t_R + \frac{M_\perp c}{QE\beta}\left\{\left[\frac{1}{\gamma} - \gamma_R(1-\beta\beta_R)\right]\frac{\Delta\left(\frac{QE}{M_\perp}\right)}{\frac{QE}{M_\perp}} - (\beta-\beta_R)\Delta(\gamma\beta)_R\right\} \tag{4}$$

For the values $M_\perp = 70\ m_e$, QE = 50 kv/cm, $\beta \sim 1$, $\frac{1}{\gamma} \sim 0$, $\beta = \frac{1}{2}$,

$$\Delta t = \Delta \tau_R - 10^{-8}\left[\frac{\Delta\left(\frac{QE}{M_\perp}\right)}{\frac{QE}{M_\perp}} + \frac{1}{2}\ \frac{\Delta\beta_R}{\beta_R}\right]\ (\text{seconds}) \tag{5}$$

It would thus appear that variations of as much as 10% in the parameters of the released ring would not affect the timing, provided that the pulses are long enough to cover the anticipated range of 5-10 nano-seconds in Δt_R. It should be remembered, however, that eq (3) shows that γ at the end of the accelerator is proportional to $(\frac{QE}{M_\perp})$, so that this quantity must be controlled to more like 1% to achieve an acceptable uniformity in final proton energy.

If the uncertainty in release time is indeed dominant and if it is both desirable and possible to keep the accelerator pulse short, the trigger for the accelerator could be provided by a signal created by the ring passing an upstream reference point far enough from the accelerator to allow for jitter. Assuming that the information as to the whereabouts of the ring can be transmitted accurately at the speed of light, the drift distance required is $L = \beta_R c \Delta t$, where $\Delta\tau$ is the uncertainty in pulsing the lines - i.e., about 15 cm per nanosecond accelerator jitter. Thus a meter or two of drift distance should be adequate; fluctuations in β_R would not seriously affect arrival time at the accelerator if the distance is that short.

C. R.F. Acceleration

An r.F. accelerator seems to present greater complications. The fields would have been established much earlier and nothing much could be done about re-phasing on this time scale without increasing the instantaneous power requirements very substantially. It would seem, therefore, that the synchronization must be achieved by acting on the ring, with the accelerator serving as time reference. A straightforward way to accomplish this end would be to precede the accelerator by a bunching cavity, tied to the accelerator in phase but oscillating at a sufficiently low sub-harmonic that the range of release-time errors falls within the linear range of the buncher sine-wave. By this means, the ring could be given a velocity increment proportional to its time deviation from ideal and thus, after drifting some distance, arrive at the accelerator at the correct phase, though with the wrong velocity. Unfortunately, the numbers look unattractive; in order to cover 10 ns uncertainty in a quarter of an oscillation requires a buncher frequency of about 25 megacycles, and the drift distance required to make up the time error, given by

$$L = \beta_R c \frac{\Delta t_R}{\left(\frac{\Delta\beta}{\beta_R}\right)} , \qquad (6)$$

is 70 meters for $\Delta t_R = 5 \times 10^{-9}$ sec, $\Delta\beta/\beta_R = 0.01$, and $\beta_R = \frac{1}{2}$. Before seriously contemplating such a scheme, it would be important to measure what Δt_R really is, and to calculate what range of velocity and phase errors is tolerable to the accelerator. Such a tolerance calculation is not as easy to do as for the pulsed line accelerator but rather necessitates an integration of the phase oscillation equation to determine the final γ of the ring as a function of initial conditions. For the present, the tolerance of 1% on velocity used above is nothing more than a guess.

It might be possible to correct the arrival times of the ring without introducing a velocity error by using a magnetic perturbation. The time of passing a pick-up loop shortly following the compressor could be compared to the phase of the accelerator and the magnetic field between compressor and accelerator could be raised or lowered to change the ring velocity and thus the time of arrival at the accelerator. Expression (6) applies to this situation also, but now Δt_R need be no more than half an r.f. cycle and the fractional velocity change in the altered field region could be large, since it is undone when the ring encounters normal field strength again. In this case, the drift length is probably determined by the time needed to make the field change before the arrival of the ring. From eq'n (1)

$$\frac{\beta_{Ra}}{\beta_R} = \sqrt{\frac{B_{cp}-B_{Ra}}{B_{cp}-B_R}} \sim 1 + \frac{1}{2}\,\frac{B_R - B_{Ra}}{B_{cp}-B_R} \qquad (7)$$

where the extra subscript, a, refers to values in the altered section of solenoid. If B_{cp} = 20 kg, B_R = 15 kg, β_R = 0.5, and β_{Ra} differs from β_R by 10%, then $B_R - B_{Ra}$ = 1 kg, a field difference which must be established over a distance of a meter or so before the ring arrives. For a rate of rise of 10^9 gauss for the correcting supply, the required drift distance would be 150 meters.

D. Conclusion

These numbers can be juggled considerably, and this report is only intended as an attempt to point out some of the problems. As mentioned in section (A), the nominal release velocity must not be too low, in the interest of reproducibility; on the other hand, the above examples show that the correcting distances can easily get out of hand if it is too high. In the case of an r.F. accelerator, the drift times should be small compared to an r.F. period divided by the uncertainty in release velocity if a timing correction is to succeed. Measurements of ring velocity and $\frac{Q}{N_1}$ have not been considered because the author has no idea of the achievable accuracy.

Appendix On the Validity of Adiabatic Formulas

The formulas used here and for the peristaltic process during acceleration for the effect of spatial variations in the solenoidal magnetic field are based on the assumption that the field changes take place over many rotation periods of the electrons. In fact, the axial distance traversed per transverse revolution increases so rapidly with $\beta_{\parallel}$ that the assumption is not very good, even for the manipulations contemplated in this note at low velocity. The axial progress per turn is given by:

$$S = 2\pi R \beta_{\parallel} \gamma_{\parallel} \tag{8}$$

which, for R = 4 cm, is already 25 cm at $\beta_{\parallel} = 0.7$.

Some idea of the seriousness of this effect can be obtained by considering the opposite extreme of a sudden step in longitudinal field. In transition, $\Delta R = \Delta p_R = 0$, but $\Delta p_{\perp} = \frac{eR}{2c} \Delta B$, so that the center of curvature of the orbit is displaced by an amount, $\frac{\Delta B}{2B}$ R. Thus, except for possible self-field effects, the ring would thereafter pulsate radially at rotation frequency with that amplitude. On the other hand, the change in axial speed is the same as in adiabatic approximation, so that one might anticipate that the desired timing and peristaltic effects do not depend on the approximation. The remaining worry would be the effect of inducing a coherent radial oscillation by imposing field changes over too short a distance.